Week 6 Lab problems

EEB 429
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Rmd note - Submit HTMLs for class HWs

A. Probabilities of composite events

Assume that the probability of snowing on any day in the winter is 0.3 (and independent of whether it snowed on other days).

1. What is the probability that it does NOT snow on two consecutive days?
2. What is the probability that it does NOT snow on 10 consecutive days?
3. What is the probability that it does NOT snow on n consecutive days?
4. What is the probability that it snows on the n^{th} day, and not on any day before that?

Let's say you have two coins (labeled C1 and C2) and two dice (labeled D1 and D2), what is the probability of each of these outcomes when you flip/roll all of them:

5. The sum of the two dice is 7 and both coins show heads.
6. Either
    a. There is exactly one head and the sum of the dice is more than 3, or
    b. There are no heads and the sum of the dice is less than 6
7. C1 is heads, C2 is either heads or tails, D1 is 3, and D2 is less than 5.

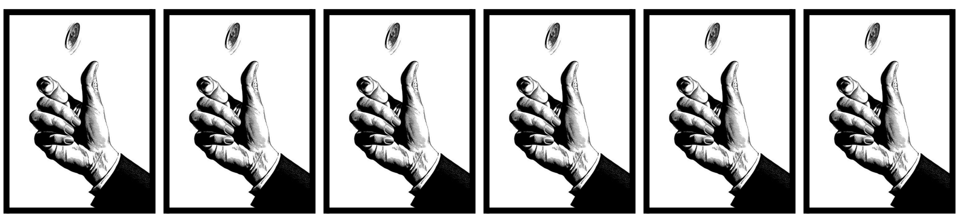
B. Conditional probabilities and independence

1. If $p(A) = 1/2, p(B) = 1/11$, and events A and B are independent, what is the probability of $p(A\ and\ B)$?
2. Now assume that A and B are not independent, why can't we use the same formula as in B.1 to get $p(A\ and\ B)$?
3. Now, if you are given $p(A|B) = 1/5$ (and the values in part B.1 hold), what is $p(A\ and\ B)$? Are A and B independent in this case?
4. Given, $p(A) = 1/2, p(B) = 1/11$, and $p(A|B) = 1/5$, what is $p(B|A)$?

C. Probability of sequences

Let us imagine a large number of monkeys on typewriters with keys labeled A to Z (total 26). The monkeys tap the keys completely randomly and the typewriter automatically stops working after six letters have been typed.

1. What is the probability that a given monkey types the word “BANANA”?
2. Hence, approximately how many monkeys in a room are required to get the word BANANA typed?



Now, instead of monkeys, imagine a large number of people flipping a **fair** coin six times each.

1. What is the probability that a given person gets the sequence “`HTTHTT`”?
2. Now, assume that the coin is **unfair,** such that a heads appears with a probability p, and thus tails appears with a probability $q = 1 - p$. What is the probability that a given person gets the sequence “`HTTHTT`”?
3. What is the probability that a given person gets the sequences “`THTHTT`” with the **unfair** coin?
4. With the unfair coin, out of 10,000 people, approximately how many will get the sequence “`HTTHTT`”? How many will get the sequence “`THTHTT`”?
5. Convince yourself that each *re-arrangement* of “`HTTHTT`” will be obtained by the same number of people as in C.4 (i.e., each rearrangement has the same probability of appearing). How many unique re-arrangements of “`HTTHTT`” are there?

   (Hint: think about *choosing* two positions for the H’s in the sequence out of six positions).

6. Hence, what is the total probability of getting 2 heads and 4 tails in the process (arranged in any order)?

"Fun" probability reading for the spring break (totally optional).