CMPLXSYS/POLSCI 391 — Winter 2022 — Discussion
Equilibrium
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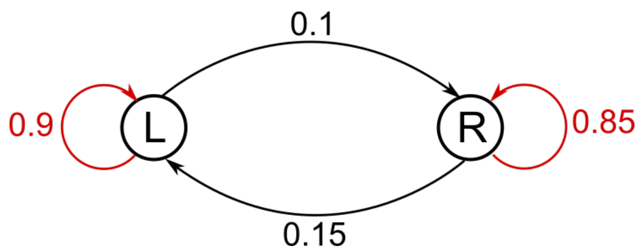
Homework

Question 1: Equilibrium in a two-state Markov model

(2 points)

Last week we wrote down a Markov model for a group of people switching between right and left political ideologies given by the following diagram.



We also wrote down the corresponding equations for the number of people in each state at time t

$$n(L)_t = n(L)_{t-1} - n(L \to R)_{t-1} + n(R \to L)_{t-1} \tag{1}$$

$$n(R)_t = n(R)_{t-1} - n(R \to L)_{t-1} + n(L \to R)_{t-1} \tag{2}$$

Assuming the transition rate given in the diagram, solve the above equations for the equilibrium number of people in the left and right states (Show your work).

(Hint: The transition terms $n(L \to R)_{t-1}$ and $n(R \to L)_{t-1}$ can be written in terms of $n(L)_{t-1}$, $n(R)_{t-1}$, and the transition rates 0.1 and 0.15)

Question 2: Equilibrium in a bakery

(1 point)

Bread from a bakery is bought by customers at a rate r_c. The bakery can regulate it's rate of production r. They realise that depending on their rate, there is either - (A) an excess of bread at the end of the day, (B) A deficiency when customers ask for bread, or (C) no excess or deficiency.

1. What are the values of r (compared to r_c) for which each of these scenarios take place?

2. Based on these observations, the bakery decides to implement a new system that regulates the rate r based on the amount of bread already present in the store. A constant rate r_p is thus scaled by B, the amount of bread already present. The rate of increase for bread in the store is now given by,

$$\text{Rate of increase for bread in store} = \frac{r_p}{B} - r_c \tag{3}$$

What is the amount of bread in the store at equilibrium? (Show your work)

Question 3: Equilibrium in an economy (Cobb-Douglas)

(1 point)

In a simplified version of the Cobb-Douglas model, the output rate (in dollars per unit time) of an economy that relies on machines (M) and labor (L) is dependent on the two factors as.

$$O \text{ (Output Rate)} = c\sqrt{L}\sqrt{M} \tag{4}$$

Some of this output is invested back into the economy as capital for buying new machines. The total investment (per unit time) is then,

$$I \text{ (Investment Rate for new machines)} = sO = sc\sqrt{L}\sqrt{M} \tag{5}$$

However, some of the machines also depreciate (become non-operational) and thus you have to spend some capital to maintain them. The rate of depreciation is given by.

$$D \text{ (Depreciation Rate for machines)} = dM \tag{6}$$

Here, s, m, and d are all constants. The overall rate of growth of number of machines can thus be given as,

$$\begin{aligned}\text{Rate of increase in machines} &= \text{Rate of investment} - \text{Rate of depreciation} \\ &= I - D = sc\sqrt{L}\sqrt{M} - dM\end{aligned}$$

Using this information, calculate the equilibrium number of machines in terms of s, c, d, and L (Show your work).

Question 4: Equilibrium in nature (Lotka-Volterra)

(1 point)

The rate of increase in predator (Y) and prey numbers (X) in a natural reserve is given by.

$$\text{Rate of prey increase} = \dot{X} = rX - cXY \tag{7}$$

$$\text{Rate of predator increase} = \dot{Y} = pXY - dY \tag{8}$$

Find the equilibrium numbers of predator and prey (Show your work). Hint: This exact problem was discussed in the lecture slides, except I've changed some of the variable names.